Design of Lattice Reduction Algorithms for Linear-Precoded MIMO System

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***Abstract*—In this letter, the design of lattice reduction (LR) algorithms specialized for improving the performance of a linear-precoded MIMO system is investigated. Conventionally, the lattice reduction algorithm required in such system is performed mostly by either the Lenstra-Lenstra-Lovász (LLL) or the Seysen's algorithm (SA), which is designed to search for shorter or nearly-orthogonal bases of a lattice. In this letter, we show that additional performance gain can be further obtained by taking the mean-square-error (MSE) of the considered MIMO system into account. The computational complexity and the error rate achieved by the proposed algorithm is quantified and compared with the LLL and SA via numerical simulations.**

***Index Terms*—MIMO, lattice-reduction, linear precoder.**

I. Introduction

MULTIPLE-input-multiple-output (MIMO) communications have drawn significant research interests in recent years. The enormous capacity provided by the MIMO channels can be exploited by spatial multiplexing, the technique of simultaneously transmitting multiple data streams via multiple spatially-separated antennas [1, 2]. To combat the spatial mixing caused by the channel, and to relax the computational burden at the receiver side, a number of precoding schemes have been proposed. Among these precoding schemes, linear precoding gains its popularity due to its simplicity for implementation, but it also suffers from significant performance loss if the channel is ill-conditioned [3].

Recently, lattice-reduction (LR) has been introduced to improve the performance of many low-complexity MIMO transceiver architectures [4]–[6]. Through lattice reduction, the original MIMO channel is transformed into an equivalent one with better condition, and hence the detection can be performed with better reliability [4, 5]. In LR-aided MIMO techniques, the Lenstra-Lenstra-Lovász (LLL) algorithm [7] has been considered almost exclusively to date, as it features average polynomial-time complexity in finding the shortest vector within an exponential factor. More recently, the Seysen's algorithm (SA) [8] has also been proposed for both LR-aided MIMO detection [9] and LR-aided MIMO precoding [10]. As the Seysen's algorithm empirically finds a basis with smaller condition number compared to the LLL algorithm for lattice dimension less than 31 [11], improved error rate performance has been observed [9, 10]. However, as the error rate of a communications system strongly depends on the transceiver architecture, the LLL and SA which are designed purely from the perspective of orthogonality may not provide the best performance. Consequently, some recent research efforts on advanced LR algorithms for LR-aided linear detection [12] and large-scale LR-aided MIMO detection [13] have started to appear in the literature, while the problem of advanced LR algorithms design for LR-aided MIMO precoding remains open, to the best of authors' knowledge.

In this letter, we investigate the design of lattice reduction algorithms specialized for a point-to-point LR-aided linear-precoded MIMO system. Unlike the LLL or SA which are designed to search for a relatively short or orthogonal basis, the proposed algorithm is designed to monotonically decrease the mean-square-error (MSE) and is guaranteed to converge to at least a local minimum. As a result, the proposed algorithm can be used to acquire additional performance gain when combined with the conventional LLL and SA.

Notations: Throughout this letter, matrices and vectors are set in boldface, with uppercase letters for matrices and lower case letters for vectors. $\|\cdot\|_{\rm F}$, $\mathrm{tr}\{\cdot\}$ and $\det\{\cdot\}$ denote the Frobenius norm, trace, and determinant of a matrix, respectively. $\mathrm{E}\{\cdot\}$ denotes the expectation operator and $\Re\{\mathbf{X}\}$ denote the real part of $\mathbf{X}$. The superscripts ^{T}, ^{H}, and † denote the transpose, conjugate transpose, and the right-inverse of a matrix respectively. For a full row rank matrix $\mathbf{A}$, $\mathbf{A}^\dagger = \mathbf{A}^H(\mathbf{A}\mathbf{A}^H)^{-1}$. $\mathbf{I}_N$ denotes the $N\times N$ identity matrix.

II. System Description

Fig. 1 shows the considered point-to-point LR-aided MIMO linear precoded system as described in [5], where perfect instantaneous channel state information is assumed to be available at the transmitter. The transmitter and the receiver are assumed to be equipped with M antennas and N antennas, respectively, where $M \geq N$. In this system, the $N\times 1$ symbol vector $\mathbf{s}$ is first transformed into the $N\times 1$ vector $\mathbf{z}$ by multiplying by a unimodular matrix $\mathbf{T}^{-H}$, and then filtered by a linear precoder $\tilde{\mathbf{F}}$. A power scaling factor β is then introduced to the precoded symbol vector such that the transmitted signal vector $\mathbf{x} = \beta\tilde{\mathbf{F}}\mathbf{T}^{-H}\mathbf{s}$ satisfies the power constraint $\mathrm{E}\{\mathrm{tr}\{\mathbf{x}\mathbf{x}^H\}\} = N\sigma_s^2$. Here we assume each symbol in $\mathbf{s}$ is drawn independently from the same rectangular quadratic-amplitude-modulation (QAM) constellation with average energy σ_s^2. Under the assumption of classic frequency flat Rayleigh fading channel, the input-output relationship of the system can then be concisely described by

$$\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{w} = \beta\mathbf{H}\mathbf{F}\mathbf{s} + \mathbf{w}, \tag{1}$$

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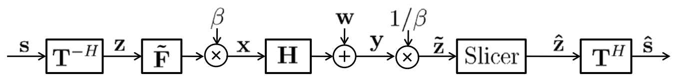

Fig. 1. A simplified block diagram for a LR-aided linear precoded MIMO system.

where $\mathbf{F} = \tilde{\mathbf{F}}\mathbf{T}^{-H}$ is the overall linear precoder, $\mathbf{y} \in \mathbb{C}^N$ is the received signal vector, $\mathbf{H} \in \mathbb{C}^{N\times M}$ is the channel matrix, and $\mathbf{w} \in \mathbb{C}^N$ denotes the noise vector modeled by a zero-mean circularly symmetric complex Gaussian vector with covariance matrix $\sigma^2\mathbf{I}_N$.

In the framework of LR-aided linear precoding, the precoded signal vector $\mathbf{Fs}$ is interpreted as a point in the lattice described by the basis matrix $\mathbf{F}$. Since changing the basis does not change the lattice, the matrix $\tilde{\mathbf{F}} = \mathbf{F}\mathbf{T}^H$ can also be used to represent the same lattice as long as $\mathbf{T}^H$ is unimodular. Usually, a basis matrix with better condition is preferred, and the overall procedure of finding such basis is referred as the lattice-reduction. In the framework of LR-aided linear precoding, $\mathbf{T}$ can be obtained by applying standard LR algorithms such as the LLL [7] or SA [8] to $\mathbf{H}^H$ under ZF criterion or $[\mathbf{H}, \sqrt{\alpha}\mathbf{I}_N]^H$ under MMSE criterion, respectively. Here $\alpha = \sigma^2/\sigma_s^2$.

With this new lattice basis $\tilde{\mathbf{F}}$, the precoded signal can then be represented as $\tilde{\mathbf{F}}\mathbf{z}$, where $\mathbf{z} = \mathbf{T}^{-H}\mathbf{s}$. An equivalent system model of (1) is then obtained as

$$\mathbf{y} = \beta\mathbf{H}\tilde{\mathbf{F}}\mathbf{z} + \mathbf{w}, \tag{2}$$

in which $\mathbf{z}$ is interpreted as the transmitted symbol vector being precoded and scaled by $\tilde{\mathbf{F}}$ and β before entering the channel. The precoder $\tilde{\mathbf{F}}$ can be designed as $\tilde{\mathbf{F}} = \mathbf{H}^\dagger$ under ZF criterion, and $\tilde{\mathbf{F}} = \mathbf{H}^H(\mathbf{H}\mathbf{H}^H + \alpha\mathbf{I}_N)^{-1}$ under MMSE criterion, respectively.

From the new equivalent model (2), the effect of β is first compensated at the receiver side before detecting $\mathbf{z}$. The result of detection, $\hat{\mathbf{z}}$, is then converted back to the original constellation via $\hat{\mathbf{s}} = \mathbf{T}^H\hat{\mathbf{z}}$. Proper rounding to the constellation boundary will be required if the elements of $\hat{\mathbf{s}}$ lie outside of the constellation set. However, this quantization effect is negligible for large QAM constellations, and can be mitigated via many existing list-detection schemes. The transformation matrix $\mathbf{T}^H$ at the receiver can be either computed at the receiver or obtained from the transmitter. Thanks to the unimodular nature of $\mathbf{T}$, the required information for $\mathbf{T}^H$ can be conveyed quite economically via a low rate control channel. The computational complexity at the receiver is also low since the elements in $\mathbf{T}^H$ are all complex integers and hence all the multipliers can be implemented economically using simple barrel-shifters.

In the following sections, we derive an new LR algorithm aiming at the minimization of the MSE for this architecture, and show by simulation that additional performance gain is obtained by exploiting the prior knowledge of this transceiver architecture.

III. Proposed lattice-reduction algorithm for improved LR-aided linear precoding

In this section, we first derive the MSE for the LR-aided ZF and MMSE linear precoded system and then propose a practical reduction algorithm which monotonically minimizes the derived MSE through successive basis updates.

A. LR-aided ZF precoding

In the LR-aided ZF precoded system shown in Fig. 1, the scaled received vector $\tilde{\mathbf{z}}$ can be expressed as

$$\tilde{\mathbf{z}} = \mathbf{y}/\beta = \left(\beta\mathbf{H}\tilde{\mathbf{F}}\mathbf{z} + \mathbf{w}\right)/\beta = \mathbf{z} + \mathbf{w}/\beta. \tag{3}$$

As a result, the error rate performance of detecting $\mathbf{z}$ is completely determined by the value of β, given by

$$\beta = \sqrt{\frac{N}{\mathrm{tr}\{\mathbf{T}^{-1}\tilde{\mathbf{F}}^H\tilde{\mathbf{F}}\mathbf{T}^{-H}\}}} = \sqrt{\frac{N}{\mathrm{tr}\{\mathbf{T}^{-1}(\mathbf{H}\mathbf{H}^H)^{-1}\mathbf{T}^{-H}\}}}. \tag{4}$$

To achieve the best error rate performance, the optimal lattice reduction algorithm should be designed such that β is maximized, or equivalently, $\mathrm{tr}\{\mathbf{T}^{-1}(\mathbf{H}\mathbf{H}^H)^{-1}\mathbf{T}^{-H}\}$ is minimized. This is equivalent to minimizing the MSE of $\tilde{\mathbf{z}}$, since

$$\begin{aligned}\mathrm{MSE} &= \mathrm{tr}\{(\tilde{\mathbf{z}} - \mathbf{z})(\tilde{\mathbf{z}} - \mathbf{z})^H\} = N\sigma^2/\beta^2 \\ &= \sigma^2\mathrm{tr}\{\mathbf{T}^{-1}(\mathbf{H}\mathbf{H}^H)^{-1}\mathbf{T}^{-H}\}.\end{aligned} \tag{5}$$

B. LR-aided MMSE precoding

In LR-aided MMSE precoded system, the scaled received vector $\tilde{\mathbf{z}}$ is given by

$$\tilde{\mathbf{z}} = \mathbf{y}/\beta = \mathbf{H}\mathbf{H}^H(\mathbf{H}\mathbf{H}^H + \alpha\mathbf{I}_N)^{-1}\mathbf{z} + \mathbf{w}/\beta, \tag{6}$$

where

$$\beta = \sqrt{\frac{N}{\|\mathbf{T}^{-1}(\mathbf{H}\mathbf{H}^H + \alpha\mathbf{I}_N)^{-1}\mathbf{H}\|_{\mathrm{F}}^2}}. \tag{7}$$

Using (6) and (7) along with some mathematical manipulations, the MSE of $\tilde{\mathbf{z}}$ can be obtained as

$$\mathrm{MSE} = \sigma^2\mathrm{tr}\{\mathbf{T}^{-1}(\mathbf{H}\mathbf{H}^H + \alpha\mathbf{I}_N)^{-1}\mathbf{T}^{-H}\}. \tag{8}$$

The MSE expressions in (5) and (8) suggest that the optimal (in the MMSE sense) LR algorithm for LR-aided linear precoding should be designed such that $\mathrm{tr}\{\mathbf{T}^{-1}\mathbf{A}\mathbf{T}^{-H}\}$ is minimized, where $\mathbf{A} = (\mathbf{H}\mathbf{H}^H)^{-1}$ for ZF and $\mathbf{A} = (\mathbf{H}\mathbf{H}^H + \alpha\mathbf{I}_N)^{-1}$ for MMSE. In the following, we propose a suboptimal greedy-search algorithm to achieve this goal.

C. Proposed LR algorithm for LR-aided linear precoding

For notational convenience, we define $\mathbf{D} = \mathbf{T}^{-H}$. Then the design objective of the lattice reduction becomes finding a unimodular matrix $\mathbf{D}$ such that $\mathrm{tr}\{\mathbf{D}^H\mathbf{A}\mathbf{D}\}$ is minimized.

Initialization: Compute $\mathbf{D} = \mathbf{T}_0^{-H}$ and $\mathbf{E} = \mathbf{D}^H\mathbf{A}\mathbf{D}$, where $\mathbf{T}_0$ is the unimodular transformation matrix obtained by some existing LR algorithms such as LLL, SA, and etc.

Iterations: Let $\mathbf{U}_{r,s}$ denote the $N \times N$ matrix with a one at the (r, s)th entry and zero elsewhere. Then for each candidate (i, j) pair in each iteration, we consider the basis update operation of the following form:

$$\begin{aligned}\mathbf{D}' &= \mathbf{D}(\mathbf{I}_N + \lambda_{i,j}\mathbf{U}_{i,j}) \\ &= [\mathbf{d}_1, \ldots, \mathbf{d}_{j-1}, \lambda_{i,j}\mathbf{d}_i + \mathbf{d}_j, \mathbf{d}_{j+1}, \ldots, \mathbf{d}_N],\ i \neq j,\end{aligned} \tag{9}$$

in which the jth column of $\mathbf{D}'$ is replaced by $\lambda_{i,j}\mathbf{d}_i + \mathbf{d}_j$ for some complex integer $\lambda_{i,j}$. Note that $\mathbf{D}'$ remains unimodular for any complex integer $\lambda_{i,j}$. The variation in the MSE corresponding to this update can be expressed as

$$\begin{aligned}\Delta(\lambda_{i,j}) &= \mathrm{tr}\{\mathbf{D}'^H\mathbf{A}\mathbf{D}'\} - \mathrm{tr}\{\mathbf{D}^H\mathbf{A}\mathbf{D}\}\\ &= 2\Re\{\lambda_{i,j}^* E_{i,j}\} + |\lambda_{i,j}|^2 E_{i,i},\end{aligned} \quad (10)$$

where $E_{i,j}$ denotes the (i,j)th element of $\mathbf{E}$. It follows that the complex integer solution minimizing (10) is obtained as

$$\hat{\lambda}_{i,j} = \left\lfloor -\frac{E_{i,j}}{E_{i,i}} \right\rceil, \quad (11)$$

where $\lfloor\cdot\rceil$ denotes the operator that rounds the argument to the nearest integer. Each iteration the algorithm computes $\hat{\lambda}_{i,j}$ and $\Delta(\hat{\lambda}_{i,j})$ among all candidate pairs (i,j), and the pair that gives the largest MSE decrease (most negative) is selected, i.e.

$$(\hat{i},\hat{j}) = \arg\min_{(i,j)} \Delta(\hat{\lambda}_{i,j}). \quad (12)$$

After $(\hat{i},\hat{j})$ is determined, the matrix $\mathbf{D}$

$$\mathbf{D} = [\mathbf{d}_1,\ldots,\mathbf{d}_{\hat{j}-1},\hat{\lambda}_{\hat{i}\hat{j}}\mathbf{d}_{\hat{i}} + \mathbf{d}_{\hat{j}},\mathbf{d}_{\hat{j}+1},\ldots,\mathbf{d}_N], \quad (13)$$

is then updated and so is the associated matrix $\mathbf{E}$.

The complexity of the proposed algorithm can be further reduced by noting (13) that only the $\hat{j}$th column in $\mathbf{D}$ is updated at the end of each iteration. Applying the definition of $\mathbf{E}$, it follows that the (r,s)th element of the updated $\mathbf{E}$ can be expressed as

$$E_{r,s} = \begin{cases} E_{r,s} & r \neq \hat{j},\ s \neq \hat{j},\\ \hat{\lambda}^*_{\hat{i},\hat{j}} E_{\hat{i},s} + E_{\hat{j},s}, & r = \hat{j},\ s \neq \hat{j},\\ \hat{\lambda}_{\hat{i},\hat{j}} E_{r,\hat{i}} + E_{r,\hat{j}}, & r \neq \hat{j},\ s = \hat{j},\\ |\hat{\lambda}_{\hat{i},\hat{j}}|^2 E_{\hat{i},\hat{i}}, +2\Re\{\lambda_{\hat{i},\hat{j}} E_{\hat{i},\hat{j}}\} + E_{\hat{j},\hat{j}}, & r = \hat{j},\ s = \hat{j}. \end{cases} \quad (14)$$

It is clear that only $2N-1$ values in $\mathbf{E}$ have to be updated.

The proposed algorithm repeats the iteration procedures until no more reduction in the MSE is possible, i.e. $\hat{\lambda}_{\hat{i},\hat{j}} = 0$. Since the MSE is bounded below from zero and is monotonically decreasing, the proposed algorithm is guaranteed to converge to at least a local minimum and hence will always improve the solution used in the initialization stage. A summary of the proposed algorithm is given in Table I

D. Complexity analysis

The complexity of the proposed algorithm can be analyzed as follows. For each candidate (i,j) pair, where $i \neq j$, the computation of $\hat{\lambda}_{i,j}$ and $\Delta(\hat{\lambda}_{i,j})$ through (11) and (10) is of complexity $\mathcal{O}(1)$. Since there are $N(N-1)/2$ different candidate pairs, the overall complexity is $\mathcal{O}(N^2)$. In (12), $(\hat{i},\hat{j})$ is determined by comparing the $\Delta(\hat{\lambda}_{i,j})$'s among all the $N(N-1)/2$ candidate pairs, and hence is also of complexity $\mathcal{O}(N^2)$. Finally, only $2N-1$ values in (14) need to be updated. Updating each value is of complexity $\mathcal{O}(1)$, and hence the overall complexity in updating $\mathbf{E}$ is $\mathcal{O}(N)$. Summing up all the required complexity in each iteration, we then conclude that except for the complexity required in the initialization, the complexity of the proposed algorithm per iteration is only of complexity $\mathcal{O}(N^2)$.

TABLE I
A SUMMARY OF THE PROPOSED LATTICE REDUCTION ALGORITHM

Input: augmented matrix $\tilde{\mathbf{H}}$. $\tilde{\mathbf{H}} = \mathbf{H}$ for ZF case, and $\tilde{\mathbf{H}} = [\mathbf{H}, \sqrt{\alpha}\mathbf{I}]$ for MMSE case.
Output: reduced matrix $\tilde{\mathbf{W}}$, unimodular matrix $\mathbf{T}$.

Initialization Stage:
Perform standard LR algorithm such as LLL or SA on $\tilde{\mathbf{H}}$ to obtain an initial unimodular matrix $\mathbf{T}_0$.
Set $\mathbf{A} = (\tilde{\mathbf{H}}\tilde{\mathbf{H}}^H)^{-1}$, and $\mathbf{D} = \mathbf{T}_0^{-H}$.
Compute $\mathbf{E} = \mathbf{D}^H\mathbf{A}\mathbf{D}$. Set flag $= 1$.

Iteration Stage:
while $flag = 1$ **do**
  for $i := 1$ to N **do**
    for $j := 1$ to N, $j \neq i$ **do**
      Compute $\hat{\lambda}_{i,j}$ as (11). Compute $\Delta(\hat{\lambda}_{i,j})$ as (10).
    end for
  end for
  Determine the selected pair $(\hat{i},\hat{j})$ using (12).
  if $\hat{\lambda}_{\hat{i},\hat{j}} = 0$ **then**
    Set $flag := 0$.
  else
    Update $\mathbf{D}$ via (13).
  end if
end while
Set $\mathbf{T} = \mathbf{D}^{-H}$ and $\tilde{\mathbf{W}} = \tilde{\mathbf{H}}^H\mathbf{T}$.

E. Relation to the $C^{(p,k)}$ algorithm and the ELR algorithm

In [12], the general $C^{(p,k)}$ algorithm has been derived, in which the metric $C^{(p,k)}(\tilde{\mathbf{H}}) = \sum_{m=1}^{M} \|\tilde{\mathbf{h}}_m^{\#}\|^{p+k}\|\tilde{\mathbf{h}}_m\|^k$ is minimized so that an approximate bound for the minimum Euclidean distance $d_{\min}$ for the LR-aided linear detector is maximized. Here $\tilde{\mathbf{h}}_m$ denotes the mth column of $\tilde{\mathbf{H}} = \mathbf{HT}$ which is the reduced matrix of $\mathbf{H}$, and $\tilde{\mathbf{h}}_m^{\#}$ denotes mth column of $\tilde{\mathbf{H}}^{\#} = \tilde{\mathbf{H}}\left(\tilde{\mathbf{H}}^H\tilde{\mathbf{H}}\right)^{-1}$. On the contrary, the MSE minimizing (minMSE) algorithm presented in this letter was designed to maximize the power scaling factor β, or equivalently to minimize the exact MSE metric for the LR-aided linear precoder.

Although the $C^{(p,k)}$ and the minMSE have been derived independently from two different objectives, it can be shown that MSE expression in (5) and (8) can be expressed as $C^{(2,0)}(\mathbf{W})$, where $\mathbf{W} = \mathbf{H}^H\mathbf{T}$ in the ZF case, and $\mathbf{W} = [\mathbf{H}, \sqrt{\alpha}\mathbf{I}_N]^H\mathbf{T}$ in the MMSE case. Since minimizing the MSE directly minimizing the bit error rate (BER), we can then conclude $(p,k) = (2,0)$ is the optimal choice among all the (p,k) pairs in the $C^{(p,k)}$ algorithm for LR-aided linear precoding. This characteristic is very different from what is observed for the LR-aided linear detection [12], where $k = 0$ together with a large p is preferred in order to maximize $d_{\min}$.

On the other hand, it is noted that while the ELR algorithm [13] is derived from the asymptotic pairwise error probability of a LR-aided linear detector and the proposed minMSE algorithm is derived from the MSE of a LR-aided linear precoder, they turn out to have similar form. This is not surprising as the close relation between the ELR and $C^{(2,0)}$ has been elaborated in [13]. However, unlike the single-stage ELR which features very low complexity for large-scale MIMO applications, a two-stage algorithm is proposed in this letter which ensures performance improvement even for problems of small dimensionality at the expense of increased complexity.

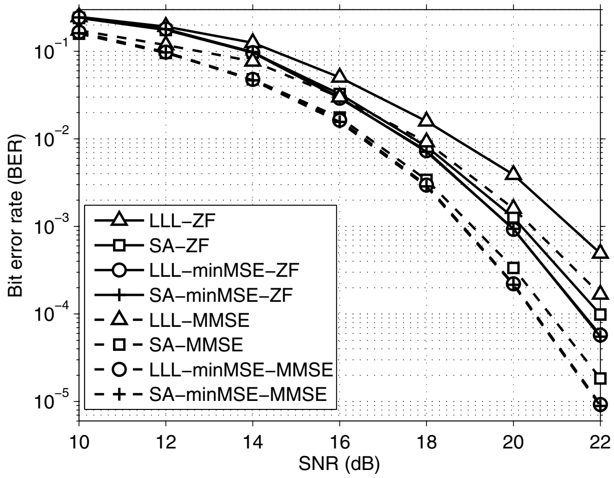

Fig. 2. BER comparison of various LR-aided linear precoders ($N = M = 8$ antennas with 16-QAM).

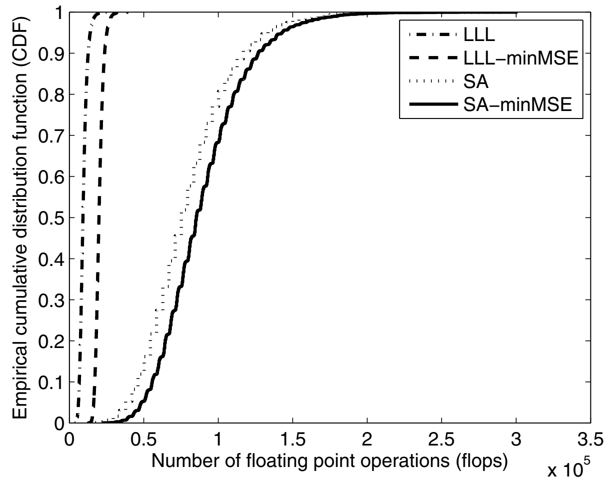

Fig. 3. Empirical CDFs of the number of floating point operations required in various lattice reduction algorithms ($N = M = 8$).

IV. Simulation

In this section, we present the simulation results of the proposed algorithm in comparison with other standard LR algorithms in the aspects of computational complexity as well as the error rate performance when applied to LR-aided linear precoding. Throughout the simulation, we assume an 8×8 MIMO system communicating over standard i.i.d. frequency-flat Rayleigh fading channel with 16-QAM constellation. We define the SNR as the average received energy per information bit divided by σ^2. Fig. 2 shows the BER performance of a number of LR-algorithms when applied to LR-aided linear precoding, where the proposed LR algorithms using LLL and SA as initialization are denoted as LLL-minMSE and SA-minMSE, respectively.

It is observed that the LLL-minMSE as well as SA-minMSE provide almost identical BER in both ZF and MMSE cases in this setting. This characteristic is very different from that of the LLL-$C^{(2,0)}$ and SA-$C^{(2,0)}$ algorithm when applied to the LR-aided linear detection [12]. This is because in LR-aided linear detector, the BER is dominated by the worst MSE among all the spatial streams, while in LR-aided linear precoding, the BER is completely determined by the sum-MSE of all spatial streams. Due to this averaging effect in the sum-MSE, the performance improvement on the worst MSE (due to the effect of lattice reduction) is less prominent on the BER of LR-aided linear precoding compared to that of LR-aided linear detection. The additional 1.6 dB gain over LLL and 0.3 dB gain over SA in the simulated scenario is obtained by exploiting the knowledge of the underlying transceiver architecture into the reduction algorithm, but also results in the increased computational complexity as shown in Fig. 3, where the empirical cumulative distribution functions of the number of floating point operations required in performing the reduction are presented. The simulation results suggests the LLL-minMSE stands out as a competitive solution, as it offers 1.6 dB gain over LLL but is only of roughly twice of the complexity, and 0.3 dB gain over SA with roughly one-fourth of the complexity.

V. Conclusion

In this letter, LR algorithms designs specialized for LR-aided linear precoding is addressed. The proposed algorithm explicitly takes the MSE of the system into account, and hence provides additional performance gain in comparison to other standard LR algorithms such as the LLL or SA.

References

[1] E. Telatar, "Capacity of multi-antenna Gaussian channels," *Europ. Trans. Telecommu.*, vol. 10, pp. 585–595, Nov.-Dec. 1999.

[2] G. J. Foschini, "Layered space-time architecture for wireless communication in a fading environment when using multiple antennas," *Bell Labs Tech. J.*, vol. 1, pp. 41–59, 1996.

[3] R. F. H. Fischer, *Precoding and Signal Shaping for Digital Transmission*. Wiley, 2002.

[4] H. Yao and G. W. Wornell, "Lattice-reduction-aided detectors for MIMO communication systems," in *Proc. 2002 IEEE Conf. Global Commun.*, pp. 424–428.

[5] C. Windpassinger and R. F. H. Fischer, "Low-complexity near-maximum-likelihood detection and precoding for MIMO systems using lattice reduction," in *Proc. 2003 IEEE Information Theory Workshop*, pp. 345–348.

[6] C. Windpassinger, R. F. H. Fischer, and J. B. Huber, "Lattice-reduction-aided broadcast precoding," *IEEE Trans. Commun.*, vol. 52, pp. 2057–2060, Dec. 2004.

[7] Jr. A. K. Lenstra, H. W. Lenstra, and L. Lovász, "Factoring polynomials with rational coefficients," *Math. Ann.*, vol. 261, pp. 515–534, 1982.

[8] M. Seysen, "Simultaneous reduction of a lattice basis and its reciprocal basis," *Combinatorica*, vol. 13, pp. 363–376, 1993.

[9] D. Seethaler, G. Matz, and F. Hlawatsch, "Low complexity MIMO data detection using Seysen's lattice reduction algorithm," in *Proc. 2007 IEEE Int. Conf. Acoust., Speech, and Signal Process.*, pp. 53–56.

[10] H. An, M. Mohaisen, and K. Chang, "Lattice reduction aided precoding for multiuser MIMO using Seysen's algorithm," in *Proc. 2009 IEEE Int. Symposium on Personal, Indoor and Mobile Radio Communications*, pp. 2479–2483.

[11] B. A. LaMacchia, "Basis reduction algorithms and subset sum problems," M.S. thesis, Massachusetts Institute of Technology, 1991.

[12] C.-E. Chen and W.-H. Sheen, "A new lattice reduction algorithm for LR-aided MIMO linear detection," *IEEE Trans. Wireless Commun.*, vol. 10, no. 8, pp. 2417–2422, Aug. 2011.

[13] Q. Zhou and X. Ma, "Element-based lattice reduction algorithms for large MIMO detection," *IEEE J. Sel. Areas Commun.*, vol. 31, no. 2, pp. 274–286, Feb. 2013.